

# An Algorithm for Robust Noninteracting Control of Ship Propulsion System

Young-Bok Kim\*

*Department of Marine Engineering, Gyeongsang National University*

In this paper, a new algorithm for noninteracting control system design is proposed and applied to ship propulsion system control. For example, if a ship diesel engine is operated by consolidated control with controllable pitch propeller (CPP), the minimum fuel consumption is achieved satisfying the demanded ship speed. For this, it is necessary that the ship is operated on the ideal operating line which satisfies the minimum fuel consumption, and the both pitch angle of CPP and throttle valve angle are controlled simultaneously. In this context of view, this paper gives a controller design method for a ship propulsion system with CPP based on noninteracting control theory. Where, linear matrix inequality (LMI) approach is introduced for the control system design to satisfy the given  $H_\infty$  constraint in the presence of physical parameter perturbation and disturbance input. To the end, the validity and applicability of this approach are illustrated by the simulation in the all operating ranges.

**Key Words** : Noninteracting Control, Controllable Pitch Propeller, Linear Matrix Inequality, Parameter Perturbation, Operating Ranges

## 1. Introduction

Due to the recent technical development in the marine industry, diesel engine propulsion system is a subject of renewed interest. Moreover, it is shown that the propulsion system control can have a significant effect on the fuel efficiency (Hendricks et al., 1986). In this context of view, this paper gives a control system design method for a marine diesel engine propulsion system with controllable pitch propeller (CPP).

In the ship propulsion system with CPP, there are many operating points which keep the ship speed constant. In order to achieve the minimum fuel consumption demand, the ship propulsion system needs to operate on the ideal line. That the two controlled outputs, engine-speed and CPP pitch angle are needed to be controlled simultane-

ously, which is difficult, because each input affects the outputs. Therefore the system used to be controlled by the conventional approach in which the one mode of two is controlled and the other is fixed. In order to overcome this problem, it is necessary that the controlled system is divided into two single-input single-output subsystems. Then each input affects only corresponding output. This paper deals with noninteracting control of the controlled system which has two independent inputs and outputs, where the inputs are fuel rack position reference signal and voltage signal of the CPP actuator and the outputs are engine-speed and CPP pitch angle. In this study, LMI approach (Boyd et al., 1990 ; Gahinet and Apkarian, 1994 ; Iwasaki and Skelton, 1993) is used to achieve  $H_\infty$  constraint in the all operating ranges. The vessel under consideration is the training ship of Pukyong National Univ. (G/T 653).

This paper is organized as follows. In Sec. 2, a dynamic model is developed for the ship propulsion system. In Sec. 3, noninteracting control system design method is introduced. Section 4

\* E-mail : kjiwoo@channeli.co.kr

TEL : +82-557-640-3162 ; FAX : +82-557-640-3128  
Dept. of Marine Engineering, College of Marine Science, Gyeongsang National University, Korea (Manuscript Received June 28, 1999; Revised January 22, 2000)

describes the constraints for control system design problem. In Sec. 5, an algorithm is proposed such that the constraints are achieved. In Sec. 6, the validity and applicability of this approach is demonstrated by the simulation. Finally, some conclusions are given in Sec. 7.

## 2. System Modeling and Control Objectives

### 2.1 Modeling

The block diagram of the controlled system is shown in Fig. 1. The inputs are the engine throttle valve angle  $R$  [rad] and the reference input of CPP actuator  $\tilde{\theta}_{cp}$  [V] to adjust CPP pitch angle.

**Table 1** Parameters

$L$ [s]	Delay time of engine
$T$ [s]	Time constant
$R$ [rad]	Engine throttle valve angle
$J$ [kg m <sup>2</sup> ]	Total inertia
$T_L$ [Nm]	Load torque
$T_D$ [Nm]	Disturbance
$n_e$ [1/s]	Engine-speed
$f$ [kg m <sup>2</sup> /s]	Friction
$V_a$ [m/s]	Ship speed
$T_s$ [s]	Time constant of CPP actuator
$K$ [rad/V]	Proportional gain

The controlled outputs are engine-speed  $n_e$  [1/s] and CPP pitch angle  $\theta_{cp}$  [rad].

The parameters appeared in Fig. 1 are summarized in the Table. The function  $e^{-Ls}$  is a time delay which is approximated with a first-order rational function. In this paper, we consider

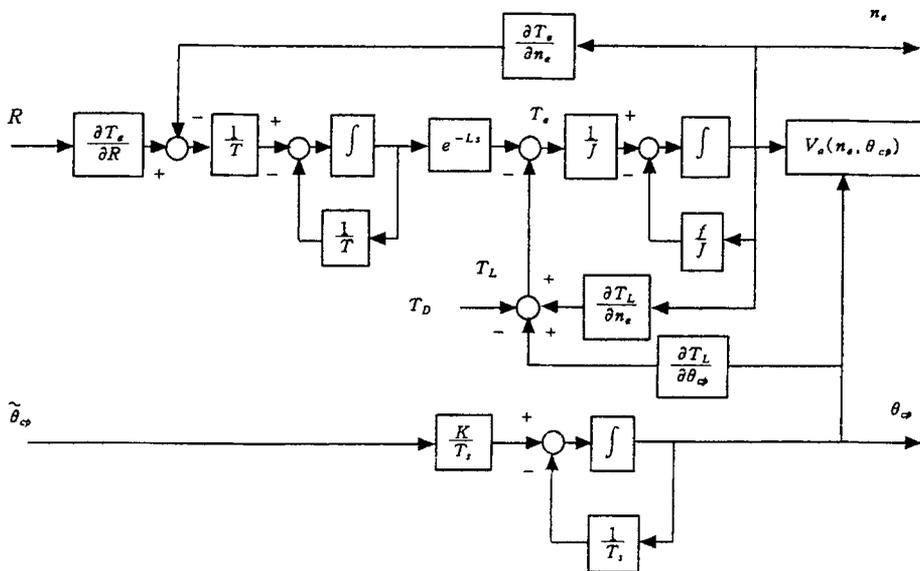
$$T, L, \frac{\partial T_e}{\partial R}, J, \frac{\partial T_e}{\partial n_e}, \frac{\partial T_L}{\partial n_e}, T_s$$

as the nonlinear terms which are linearized at each operating point. From these, we can obtain the following system representation :

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) : A(n \times n), \\ & B(n \times m) \\ y(t) &= Cx(t) : C(p \times n), \end{aligned} \quad (1)$$

where  $x(t) (= [n_e \ T_e \ \dot{T}_e \ \theta_{cp}]^T)$ ,  $u(t)$  and  $y(t)$  are the state, control input and controlled output, respectively. The coefficient matrices of the plant are denoted by

$$A = \begin{bmatrix} -\frac{1}{J} \left( \frac{\partial T_L}{\partial n_e} + f \right) & \frac{1}{J} & -\frac{1}{J} & \frac{1}{J} \frac{\partial T_L}{\partial \theta_{cp}} \\ 0 & \frac{2}{L} & \frac{4}{L} & 0 \\ -\frac{1}{T} \frac{\partial T_e}{\partial n_e} & 0 & -\frac{1}{T} & 0 \\ 0 & 0 & 0 & -\frac{1}{T_s} \end{bmatrix}$$



**Fig. 1** Block diagram of controlled system

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{T} \frac{\partial T_e}{\partial R} & 0 \\ 0 & \frac{K}{T_s} \end{bmatrix}, C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \quad (2)$$

**2.2 Control Objectives**

**2.2.1 Noninteraction between inputs and outputs**

As described in Fig. 1, the variation of CPP pitch angle disturbs the engine-speed. Therefore, by applying the noninteracting control theory, we can make the system to be decoupled into two single input single-output subsystems.

**2.2.2  $H_\infty$  Constraint**

In order to reject the steady-state tracking error for constant reference signals, the integral compensator is introduced. Consider the integral type servosystem of Fig. 2 (Fujisaki and Ikeda, 1992). Then the servosystem (augmented system) is represented by

$$\begin{aligned} \dot{\bar{x}}(t) &= \bar{A}\bar{x}(t) + \bar{B}\bar{u}(t) + \bar{B}_w w(t), \\ \bar{y}(t) &= \bar{C}\bar{x}(t) + \bar{D}w(t), \end{aligned} \quad (3)$$

where  $\bar{x}(t) = [x(t) \ v(t)]^T$ ,  $\bar{u}(t)$ ,  $w(t)$ ,  $v(t)$  are new state, control input, disturbance input and output of integrator in the augmented system, respectively. The system matrices are denoted by

$$\begin{aligned} \bar{A} &= \begin{bmatrix} A & 0 \\ -C & 0 \end{bmatrix}, \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix}, \bar{C} = [C \ 0], \\ \bar{B}_w &= \begin{bmatrix} 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}^T, \bar{D} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \end{aligned} \quad (4)$$

It is assumed that the parameters of controlled system are perturbed in the specified ranges.

For this system, it is needed to guarantee that

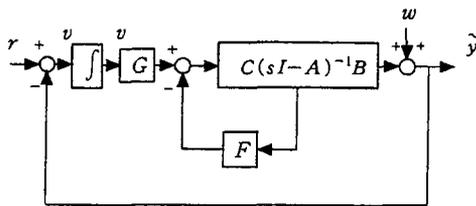


Fig. 2 An integral type servosystem

the closed-loop system is stable and RMS gain ( $H_\infty$  norm) from  $w$  to  $\bar{y}$  does not exceed  $\gamma (> 0)$ .

**3. Noninteracting Control**

Consider the system illustrated in Fig. 3. It is represented by the following linear state equation.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) : A(n \times n), \\ & \quad B(n \times m) \\ y(t) &= Cx(t) : C(p \times n) \end{aligned} \quad (5)$$

As shown in Fig. 3, linear state feedback replaces the plant input  $u(t)$  by the following expression

$$\begin{aligned} u(t) &= -Fx(t) + Gv(t) : F(m \times n), \\ & \quad G(m \times m) \end{aligned} \quad (6)$$

where,  $v(t)$  is  $m \times 1$  input signal. Then the closed-loop state equation is described by

$$\begin{aligned} \dot{x}(t) &= (A - BF)x(t) + BGv(t) \\ y(t) &= Cx(t) \end{aligned} \quad (7)$$

Noninteracting control problem involves using linear state feedback to achieve two input-output objectives. The closed-loop state (7) should be such that for  $i \neq j$  the  $j^{th}$ -input component  $v_j(t)$  has no effect on the  $i^{th}$ -output component  $y_i(t)$ . This problem is equivalent to the requirement that the closed-loop impulse response:

$$\Sigma = C \cdot \Phi_{A-BF} \cdot B \cdot G \quad (8)$$

be a diagonal matrix. A closed-loop state equation with this property can be viewed from the input-output perspective as a collection of  $m$  independent, single-input, single-output linear systems.

Here, let

$$C = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} : c_i (1 \times m)$$

for each

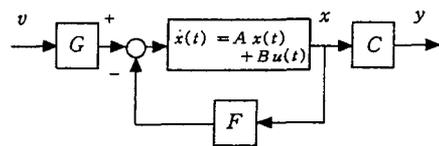


Fig. 3 Structure of linear state feedback

$$x_i = \min\{z | c_i A^{x_i-1} B \neq 0\}.$$

**Lemma 1** (Choi, 1998; Falb and Wolovich., 1967; Freund, 1971; Porter, 1969) Suppose the time-invariant linear state equation (5) with  $p=m$  has relative degree  $\chi_1, \dots, \chi_m$ . Then there exist constant feedback gains  $F$  and  $G$  that achieve noninteracting control if and only if

$$\Delta = \begin{bmatrix} c_1 A^{x_1-1} B \\ \vdots \\ c_m A^{x_m-1} B \end{bmatrix} \quad (9)$$

is invertible.

Then a pair of  $(F, G)$  is given by

$$F = \Delta^{-1} A^*, \quad G = \Delta^{-1} \quad (10)$$

where

$$A^* = \begin{bmatrix} c_1 A^{x_1} \\ c_2 A^{x_2} \\ \vdots \\ c_m A^{x_m} \end{bmatrix} \quad (11)$$

Even though the aforementioned condition is satisfied such that the noninteracting control is achieved, the system stability is not guaranteed. For this, the following lemma is considered.

**Lemma 2** (Morse and Wonham, 1971) If a pair  $(F, G)$  is given by

$$F = \Delta^{-1} [A^* + \bar{F} S_a], \quad G = \Delta^{-1} \quad (12)$$

then the noninteracting control system is stable. Where  $\bar{F}$  is a feedback gain such that the closed-loop system is asymptotically stable, where  $S_a$  is given by the following nonsingular matrix:

$$S_a = \begin{bmatrix} c_1 \\ c_1 A \\ \vdots \\ c_1 A^{x_1-1} \\ \hline c_m \\ c_m A \\ \vdots \\ c_m A^{x_m-1} \end{bmatrix} : (n \times n). \quad (13)$$

Suppose that the noninteracting condition illustrated in Sec. 3 is satisfied for the controlled system considered in this study. If the degree of

state equation does not equal to that of transfer function, then the matrix  $S_a$  is singular. Therefore, the following new matrix is introduced

$$S_b = \begin{bmatrix} S_a \\ W \end{bmatrix} \quad (14)$$

such that  $|S_b| \neq 0$ , where  $W$  is a matrix satisfying

$$W B_p = 0. \quad (15)$$

### 4. $H_\infty$ Constraint

In this paper, it is considered that system parameters are varying in the specified ranges. For this system, robust stability and  $H_\infty$  performance problems of the servosystem are considered. These problems are given by following Theorem.

**Theorem** For the system described by Eq. (3), the  $H_\infty$  norm of  $T_{yw}$  which is the closed-loop transfer function of the system (3) via state feedback (6) is smaller than  $\gamma (> 0)$ , if and only if there exist  $X (> 0)$  and parameter  $Y$  satisfying the following LMI:

$$\begin{bmatrix} \bar{A}X + X\bar{A}^T + \bar{B}Y + Y^T\bar{B}^T & \bar{B}_w & X\bar{C}^T \\ & \bar{B}_w^T & -\gamma I & \bar{D}^T \\ & \bar{C}X & \bar{D} & -\gamma I \end{bmatrix} < 0 \quad (16)$$

Then the feedback gain  $K_{FG}$  is given by

$$K_{FG} = YX^{-1} = [F \ G] \quad (17)$$

**Proof** See reference (Gahinet and Apkarian, 1994)

This result is easily extended to uncertain systems described by polytopic state-space model.

Let us denote matrices  $\bar{A}, \bar{B}$  as

$$\bar{A} = \bar{A}_m + \delta\bar{A}, \quad \bar{B} = \bar{B}_m + \delta\bar{B}, \quad (18)$$

where  $\bar{A}_m, \bar{B}_m$  are nominal parts and  $\delta\bar{A}, \delta\bar{B}$  denote uncertain parts.

Here, a standard numerical method is considered to check whether (16) holds or not. If the set of uncertain plant is polytopic, that is, the set described as

$$\bar{A}(a) = \bar{A}_m + \delta\bar{A}(a) = \bar{A}_m + \sum_{i=1}^k \alpha_i \bar{A}_i,$$

$$\begin{aligned} \bar{B}(\alpha) &= \bar{B}_m + \delta \bar{B}(\alpha) = \bar{B}_m + \sum_{i=1}^k \alpha_i \bar{B}_i, \\ \sum_{i=1}^k \alpha_i &= 1, \alpha_i \geq 0, i=1, \dots, k \end{aligned} \quad (19)$$

then we can easily find the minimum  $\gamma$ , a positive definite matrix  $X$  and a matrix  $Y$  satisfying the condition (12) and (16) at all extreme points, simultaneously.  $(\bar{A}_i, \bar{B}_i)$ ,  $i=1, 2, \dots, k$  denote the extreme points.

### 5. Algorithm

The controller design problem considered in this paper is equivalent to finding a state feedback gain  $K_{FG}(F, G)$  which satisfies the following specifications simultaneously.

- A1) Find  $K_{FG}$  satisfying the noninteracting control constraints (Lemma 1, Lemma 2) for a nominal system.
- A2) The inequality (16) of the Theorem holds.

Also, this problem can be described by the following algorithm.

**[Algorithm]**

Consider the system (1). For given sufficiently large positive number  $\gamma_n$ , set the numerical tolerance  $\epsilon > 0$  and  $n=0$ .

Step 0. Find a feedback gain  $F$  and  $G$  satisfying Lemmas 1 and 2.

Step 1. Using the relation

$$\begin{aligned} K_n &= [F_n \ G_n] = Y_n X_n^{-1}, \\ Y_n &= [F_n \ G_n] X_n \text{ and} \end{aligned}$$

solve the inequality (16) in Theorem, to get  $(X_n, \gamma_{n+1})$ .

Step 2. If  $|\gamma_n - \gamma_{n+1}| > \epsilon$ , then let  $n = n + 1$ , and go to step 0.

Otherwise, output  $(X_n, Y_n, \gamma_{n+1})$ .

Step 3. The controller gain is

$$K_{FG} = Y_n X_n^{-1} = [F_n \ G_n].$$

This problem is easily solved by LMI approach.

### 6. Simulations

In this section, we present simulation results to

illustrate the validity and applicability of the approach studied in this paper.

First, a type of controller gain for noninteracting control is calculated based on Lemmas 1 and 2. Next, check the  $H_\infty$  constraint for the servosystem with noninteracting controller in the presence of parameter perturbation. For this, consider that the parameters are varying in the following ranges:

$$\begin{aligned} 39.49 \leq \partial T_e / \partial R &\leq 114.39, \quad 0.03 \leq T \leq 0.10 \\ 0.05 \leq L &\leq 0.20, \quad 147.33 \leq J \leq 199.55, \\ 167.35 \leq \partial T_e / \partial n_e &\leq 262.04, \quad 0.05 \leq T_s \leq 0.23, \end{aligned} \quad (20)$$

$$\begin{aligned} 12585.13 \leq \partial T_L / \partial \theta_{cp} &\leq 62931.41 \\ 32.62 \leq \partial T_L / \partial n_e &\leq 944.67, \end{aligned}$$

The nominal part  $\bar{A}_m, \bar{B}_m$  of (19) are described by

$$\begin{aligned} \bar{A}_m &= \begin{bmatrix} -3.289 & 0.006 & -0.006 & 25.080 & 0 & 0 \\ 0 & -25.000 & 50.000 & 0 & 0 & 0 \\ -5204.085 & 0 & -21.665 & 0 & 0 & 0 \\ 0 & 0 & 0 & -12.175 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \\ \bar{B}_m &= \begin{bmatrix} 0 & 0 & 2103.760 & 0 & 0 & 0 \\ 0 & 0 & 0 & 12.175 & 0 & 0 \end{bmatrix}^T. \end{aligned} \quad (21)$$

These are mean values of the system matrices set.

Then,  $\bar{A}, \bar{B}$  of (3), the system matrices with uncertainties are represented by

$$\tilde{A}(\alpha) = \bar{A}_m + \sum_{i=1}^{32} \alpha_i \bar{A}_i \quad (22)$$

$$\tilde{B}(\alpha) = \bar{B}_m + \sum_{i=1}^{32} \alpha_i \bar{B}_i \quad (23)$$

$$\sum_{i=1}^k \alpha_i = 1, \alpha_i \geq 0 \quad (i=1, \dots, 32). \quad (24)$$

Based on Lemmas 1 and 2, a gain  $\bar{F}$  illustrated in (12) is obtained by

$$\bar{F} = \begin{bmatrix} f_1 & f_2 & 0 & 0 \\ 0 & 0 & f_3 & 0 \end{bmatrix}, \quad (25)$$

where  $W = [0 \ -1 \ 0 \ 0]$  (in Eqs. (14) and (15)), such that the noninteracting control system is stable. Here,  $A^*$  of equation (9) and  $\Delta$  of equation (11) are calculated from nominal values.

Even though the gain  $(F, G)$  is obtained so that the noninteracting control system is stable, the system stability may not be guaranteed for the

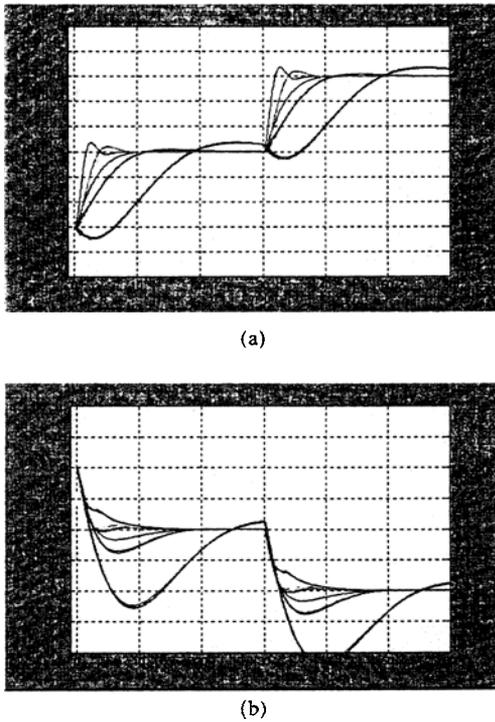


Fig. 4 Step responses without noninteracting control [(a)  $n_e$ [rps], (b)  $\theta_{cp}$ [rad)]

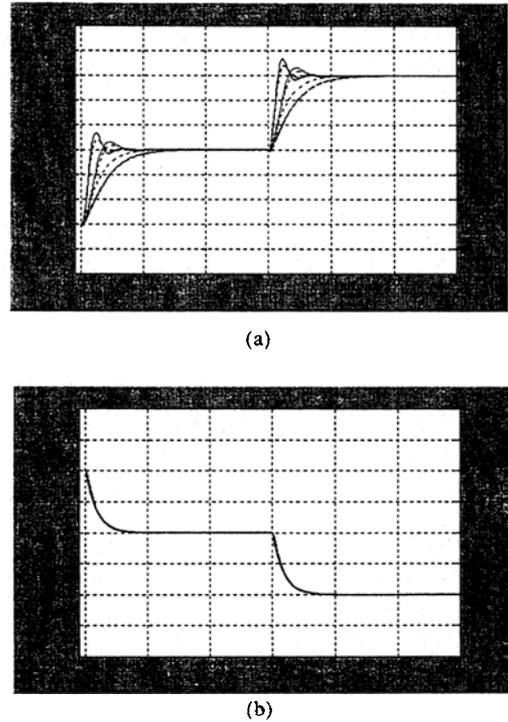


Fig. 5 Step responses with noninteracting control [(a)  $n_e$ [rps], (b)  $\theta_{cp}$ [rad)]

uncertainty. Therefore the robust stability condition represented in the Theorem for the uncertain system is considered. If we use the LMI approach, we can obtain a gain ( $F$ ,  $G$ ) based on the Algorithm proposed in Section 5.

A state feedback gain  $F$  and an integral compensator gain  $G$  are calculated as follows:

$$F = \begin{bmatrix} -0.0802 & 0.0004 & -0.0015 & 0.1369 \\ 0 & 0 & 0 & -0.0416 \end{bmatrix}$$

$$G = \begin{bmatrix} 3.2111 & -2.6163 \\ 0 & 0.5310 \end{bmatrix} \quad (26)$$

where  $f_1=1.300$ ,  $f_2=10.000$ ,  $f_3=3.000$  of  $\bar{F}$  in (25) and the bound  $\gamma=21.3$ .

Using the gain illustrated in (26), the simulation results, which are obtained in the 16 extreme points when the parameter perturbation is considered, are given in the Figs. 4~7. Especially, in the cases of Figs. 4 and 5, engine-speed and CPP pitch angle reference signals are changing at the same time as shown in the simulation results.

Figure 4 shows the controlled output to the step type reference signals when only the robust con-

trol constraint is considered without noninteracting control. In this case, following gains are used as the feedback gain.

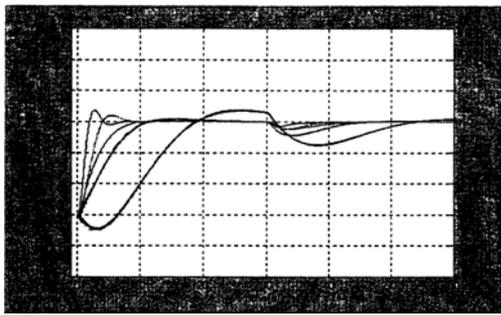
$$\tilde{F} = \begin{bmatrix} 0.0045 \times 10^{-5} & -0.0047 \times 10^{-5} & 0.2448 \times 10^{-5} \\ -0.0268 \times 10^{-5} & 0.0012 \times 10^{-5} & -0.4899 \times 10^{-5} \\ -0.0345 \times 10^{-5} \\ 0.0086 \times 10^{-5} \end{bmatrix}$$

$$\tilde{G} = \begin{bmatrix} 3.3606 & 2.2650 \\ -0.0126 & 0.2250 \end{bmatrix} \quad (26)$$

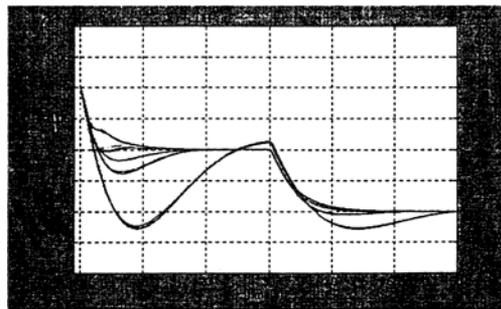
And, Fig. 5 shows the controlled output when both the noninteracting and robust control are considered. Figures 6 and 7 illustrate the controlled output and control input. In the cases of Figs. 6 and 7, the engine-speed is fixed and the CPP pitch angle is varied. From these results, it is clear that we can achieve good suppression of the interaction between inputs and outputs, and satisfy  $H_\infty$  constraint simultaneously.

### 7. Concluding Remarks

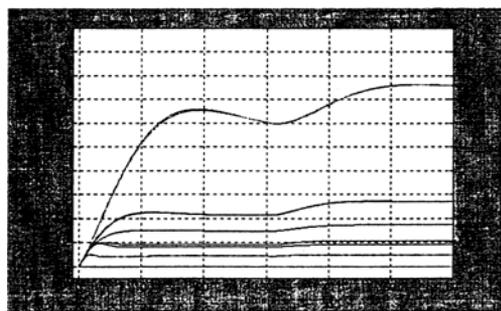
In this paper, a controller design method for a



(a)



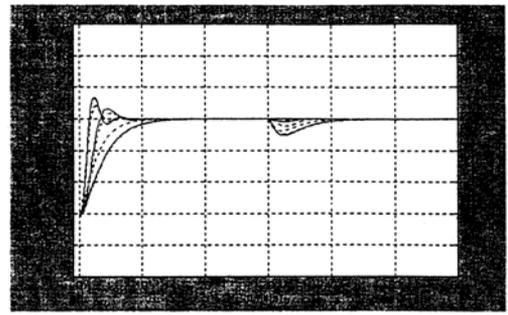
(b)



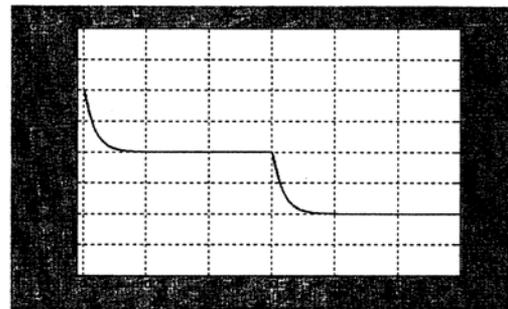
(c)

**Fig. 6** Time responses without noninteracting control when the engine-speed is fixed and the pitch angle is changing  
 [(a)  $n_e$ [rps], (b)  $\theta_{cp}$ [rad],  
 (c) control inputs]

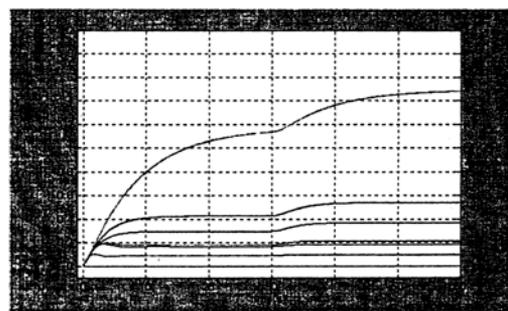
noninteracting control system design has been presented and it is applied to ship propulsion system control problem. In order to reject the interaction between inputs and outputs, the noninteracting control theory is used. Linear matrix inequality(LMI) approach is introduced so that the control system satisfies the given  $H_\infty$  constraint in the presence of physical parameter perturbation. We have shown the validity and



(a)



(b)



(c)

**Fig. 7** Time responses with noninteracting control when the engine-speed is fixed and the pitch angle is changing  
 [(a)  $n_e$ [rps], (b)  $\theta_{cp}$ [rad],  
 (c) control inputs]

applicability of this approach by achieving two given objectives simultaneously in the simulation.

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